Reliability-Based Optimization for Electromagnetic Design Employing Reliability Index Approach

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Abstract — A reliability-based optimization method for electromagnetic design is presented to take uncertainties of design parameters into account. The method can give designers the confidence of product reliability at a given probabilistic level. To achieve the goal, the reliability index approach based on the first-order reliability method is adopted to deal with probabilistic constraint functions. The proposed method is applied to the TEAM Workshop Problem 22 and its accuracy and efficiency is verified with Monte Carlo simulation result.

I. INTRODUCTION

Due to a growing demand for high-performance and high-reliability electromagnetic devices or equipment, attention has recently focused on dealing with uncertain design parameters such as manufacturing errors, operating conditions, material properties, etc [1]. The methodology for treating optimization problems in the present of uncertainties of design parameters can be categorized into two approaches. The first is robust design optimization (RDO) to improve the product quality by minimizing variability of the output performance functions; the second is reliability-based design optimization (RBDO) to achieve product reliability at a given probabilistic level. Until now, most of the reported attempts for electromagnetic design fall into RDO. Thus it is very hard to find out published articles regarding RBDO for electromagnetic applications.

In this paper, RBDO based on the first-order reliability methods (FORM) applicable to electromagnetic devices is presented. The method estimates the probability of failure (i.e. a nominal design point dose not satisfy the performance condition given) by the first-order Taylor series approximation of the performance function when probabilistic information of random variables is known [2]. To efficiently calculate the probability of failure expressed in terms of the multiple integrations of probability density functions of random parameters, the reliability index approach (RIA) is employed. The proposed method is successfully applied to the TEAM Workshop Problem 22 and its accuracy and efficiency is verified with Monte Carlo simulation result.

II. RELIABILITY-BASED DESIGN OPTIMIZATION

A. Definition of RBDO Model

In system parameter design, the RBDO model is generally defined as

$$\begin{aligned} &\text{minimize} && f(\mathbf{d}) \\ &\textit{subject to} && P(G_i(\mathbf{X}\le 0)-\Phi(\beta_{t_i})\le 0 \quad i=1,2,\dots np \\ & && \mathbf{d}^L\le\mathbf{d}\le\mathbf{d}^U,\quad \mathbf{d}\in R^n \end{aligned} \tag{1}$$

where $f(\mathbf{d})$ is the objective/cost function, $\mathbf{d}=\mu(\mathbf{X})$ is the design vector, $\mathbf{X}$ is the random vector and the probabilistic constraints are described by the performance function G_i with its probability distribution and prescribed confidence level β_{t_i}. The statistical description of the failure of G_i is characterized by the cumulative distribution function (CDF) $F_{G_i}(0)$:

$$P(G_i(\mathbf{X})\le 0)=F_{G_i}(0)=\int_{G_i(\mathbf{X})\le 0}\int f_{\mathbf{X}}(\mathbf{x})\,d\mathbf{x} \tag{2}$$

where $f_X(\mathbf{x})$ is the joint probability density function of $\mathbf{X}$. To deal with the multiple integration of (2) effectively, the reliability analysis based on FORM requires a transformation from the original random parameter $\mathbf{X}$ to the independent and standard normal random parameter $\mathbf{U}$. That is, the constraint function $G(\mathbf{X})$ in X-space is mapped onto $G(\mathrm{T}(\mathbf{X}))\equiv G(\mathbf{U})$ in U-space.

B. First-Order Reliability Analysis in RIA

For evaluating probability of (2), the first-order safety reliability index $\beta_{s,\mathrm{FORM}}$ in RIA is obtained by formulating a sub-optimization problem with an equality constraint, which is the failure surface, as

$$\begin{aligned} &\text{minimize} && \|\mathbf{U}\| \\ &\text{subject to} && G(\mathbf{U})=0 \end{aligned} \tag{3}$$

The minimum distance point on the failure surface from the origin in U-space is called the most probable point (MPP) $\mathbf{u}^*_{G(\mathbf{U})=0}$ and the reliability index is defined by $\beta_{s,\mathrm{FORM}} = \|\mathbf{u}^*_{G(\mathbf{U})=0}\|$ (see Fig. 1). After all, the solution of (3) is corresponding to the probability of (2). For the purpose of solving (3), either an MPP search algorithm that has been specified developed for the first-order reliability analysis or a general optimization algorithm can be used. Due to its simplicity and efficiency, the Hasofer Lind and Rackwitz Fiessler (HLRF) method is here employed to perform reliability analyses in RIA.

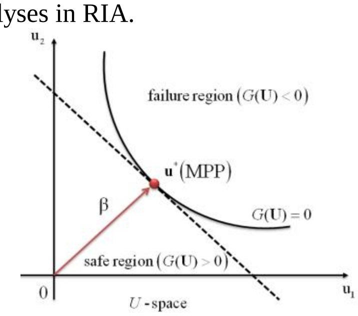

Fig. 1. Reliability index in RIA

III. NUMERICAL IMPLEMENTATION

The implementation of RBDO consists of a double-loop optimization structure as shown in Fig. 2. It means the parametric optimization problem has sub-problems for reliability analysis for each iterative design. Therefore, the procedure of the proposed RBDO problem can be divided into two optimization procedures as follows:

(a) sub-optimization procedure for failure probability of each constraint (dotted box in Fig. 2),

(b) global optimization procedure including the cost function.

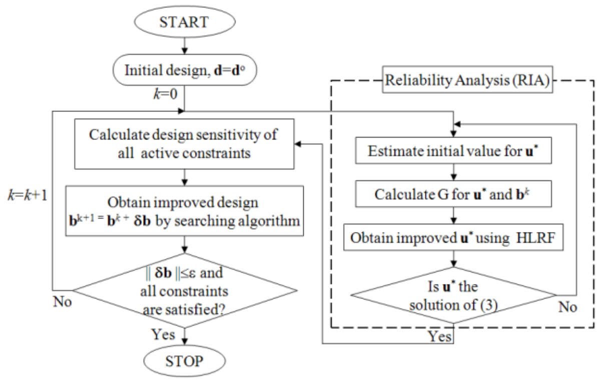

Fig. 2. Flow chart of RBOD

IV. Results

The TEAM benchmark problem 22 is concerned with RBDO of a superconducting magnetic energy storage system (SMES) as depicted in Fig. 3 [3]. In order to simplify the design problem, a constraint of the current quench condition on the superconductivity magnet is not considered and only three of total eight design variables, R^2, D^2 and H^2, are considered as independent random variables with normal probabilistic distributions. A RBDO problem for minimizing an objective function subject to a set of constraints is expressed as:

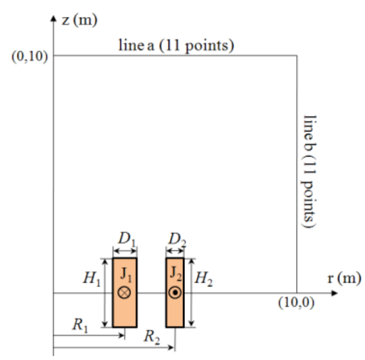

Fig. 3. Configuration of the SMES device

$$\begin{aligned} &\text{minimize} \quad f(\mathbf{d}) = \sum_{i=1}^{21} |B_{stray,i}(\mathbf{d})|^2 \\ &\text{subject to} \quad P(G_i(\mathbf{X}) \le 0) - \Phi(-\beta_{t_i}) \le 0 \quad i = 1,2 \\ &\qquad G_1(\mathbf{X}) = 1 - \left(\frac{E(\mathrm{x}) - E_o}{0.05 \times E_o}\right)^2 \\ &\qquad G_2 = (R_2 - R_1) - \frac{1}{2}(D_2 + D_1),\ \mathbf{X}^L \le \mathbf{X} \le \mathbf{X}^U \end{aligned} \tag{4}$$

where $B_{stray,i}$ is the stray field values calculated at the ith measurement point along line a and line b, E is the stored magnetic energy with a energy target value E_o of 180 MJ and the wanted confidence level β_{t_i} is set 1.645 corresponding to the failure probability value of 5 %.

The optimization problem is solved using two different optimization methods where design sensitivity values are calculated with finite differencing method (FDM). The first is a deterministic method without taking probability distributions of design variables into account; the second approach is the proposed RBDO. Starting with an initial design, the deterministic and RBDO optima are presented in Table 1 and the comparison of magnet dimensions after optimization is illustrated in Fig. 4. Results shows that the proposed method provides an optimum design satisfying the specified confidence level in the presence of uncertain parameters and produces adequate accuracy in evaluating the probability of failure (pf) is acceptable compared to Monte Carlo simulation (MCS) result.

TABLE I
DESIGN VARIABLES AND PERFORMANCE INDICATORS AT THE DETERMINISTIC AND RBDO OPTIMA

Design variables	Unit	Initial design	Deterministic optimum	RBDO optimum	Standard variation
R_1	mm	1977	1977	1977	
D_1	mm	404	404	404	
H_1	mm	1507	1507	1507	
R_2	mm	2340	2462	2348	10
D_2	mm	310	294	233	5
H_2	mm	1780	1756	1871	10
J_1	A/mm^2	16.30	16.39	16.30	
J_2	A/mm^2	16.19	16.19	16.19	
B_{stray}	μT	6,772	158	84	
Energy	MJ	179	173	178	
pf(G_1)	% (RIA)	10.1	30.2	1.2	
	% (MCS)	11.5	29.4	4.5	
pf(G_2)	% (RIA)	28.0	3.02×10^{-3}	7.12×10^{-5}	
	% (MCS)	28.0	3.02×10^{-3}	7.12×10^{-5}	

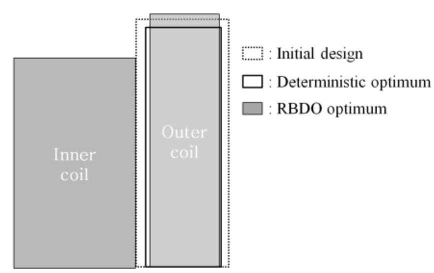

Fig. 4. Comparison of magnet dimensions after optimization

V. Conclusion

A reliability-based design optimization based on the reliability index approach has been introduced for electromagnetic design. The results reveal that the proposed method offers an acceptable design at the prescribed confidence level taking probabilistic information of random design variables into account.

VI. References

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